Week 5: Midterm Review!
MATH 4A
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Disclaimer: Since I am not the one writing the exam, I cannot guarantee this practice "exam" will look anything like the midterm. However, I reckon if you can do these without trouble, you're probably quite fine for the midterm.

Definitions: Here's a list of definitions that you should *definitely* know. Note that this list may not be comprehensive!

- A systems of equations is consistent when...?
- A matrix is in RREF when...
- A matrix is invertible when...
- Consider the set of vectors $\{v_1, v_2, \cdots, v_n\}$ in $\mathbb{R}^m$. Define the span of these vectors.
- Consider the set of vectors $\{v_1, v_2, \cdots, v_n\}$ in $\mathbb{R}^m$. We say $\{v_1, v_2, \cdots, v_n\}$ is linearly independent when...
- What does it mean for $T : \mathbb{R}^n \to \mathbb{R}^m$ to be a linear transformation?
- Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. We say T is one-to-one when... Additionally, if T is one-to-one, what can we say about m in relation to n?
- Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. We say T is onto when... Additionally, if T is onto, what can we say about m in relation to n?
- What is a basis of a vectors space?
- What is the dimension of a vector space?
- What is the nullity of a matrix? What is the null space (kernel)?
- What is the rank of a matrix? What is the column space (image)?

3-2.3 Find a set of vectors $\{u, v\}$ in $\mathbb{R}^4$ that spans the solution set of

$$\begin{cases} w - x + y - 2z &= 0, \\ 3w + 2x - y + z &= 0. \end{cases}$$

3-2.9 A $= \begin{bmatrix} -3 & 9 & -9 \\ -4 & 14 & -14 \\ 1 & -1 & 1 \end{bmatrix}$. Is it true that $Ax = b$ has a solution for every b?

4-1.5 Let $v = \begin{bmatrix} -4 \\ -6 \\ -8 \end{bmatrix}$, $u = \begin{bmatrix} -3 \\ -3 \\ 8+k \end{bmatrix}$, and $w = \begin{bmatrix} -4 \\ -1 \\ 2 \end{bmatrix}$. The set $\{v, u, w\}$ is linearly independent unless $k =$?

4-2.5 Let $v_1 = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$. Suppose $T(v_1) = \begin{bmatrix} -12 \\ 8 \end{bmatrix}$ and $T(v_2) = \begin{bmatrix} 19 \\ -9 \end{bmatrix}$. For an arbitrary vector $v = \begin{bmatrix} x \\ y \end{bmatrix}$, find $T(v)$.

5-2.12 Let $A = \begin{bmatrix} -1 & -3 & -2 \\ 1 & 3 & 2 \\ -2 & -6 & -4 \end{bmatrix}$. Find a basis for the null space (kernel) of A.